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Effects of permeability and radiation on the stability of Couette flow in a porous medium

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ABSTRACT: A study on the effects of permeability and radiation on the stability of couette flow in a porous medium was carried out. The pressure differential of the equation of motion is kept constant in the Navier-Stokes equation and solution developed by the method of undetermined coefficients.It was observed that radiation and permeability has, independently destabilizing effect on the stability of couette flow. At high wave numbers (a $\leq$ 10) and Reynolds number (Re ≥ 2500), instability sets in but at wave numbers (a ≤ 5) stability was not affected. However, within the same range of values, radiation effect was prominent. @JASEM

Couette flow results when two plates moving relative to each other cause a flow of fluid in between them. the plates could be flat, parallel or two concentric cylinders of varying radii. The stability of Couette flow problem dates back to the early investigation by Lord Rayleigh (1926) and subsequently, Taylor (1923) investigated the steady circular flow of a fluid between two rotating coaxial cylinders. In their findings, comparisons for the ratio N of 0.9 and Reynolds number (Re) up to 350, showed that an average axial velocity distribution and the exact axial distribution yield similar prediction with Taylor number. Also the Taylor number and the corresponding critical wave number differ markedly from previous narrow-gap prediction based on a parabolic approximation to the axial distribution. Chandrasekhar (1953, 1961) and Lin (1955) studied the case when the fluid is an electrical conductor and a magnetic field along the common axis of the cylinders confirmed the findings of Lord Rayleigh and Taylor that turbulence is driven by Magnetorotational instabilities.

Ng and Tuner (1981) made a numerical study of the effects of both axisymmetric and non-axisymmetric disturbances on the stability of spiral flow between rotating cylinders and found that the onset of instability depends on both the Taylor number and the axial Reynolds number. They also found out that for sufficiently high Reynolds number, there are two distinct axisymmetric modes corresponding to the usual shear and rotational instabilities and the stability boundaries for non-axisymmetric disturbance for Reynolds number (Re) ≤ 6000, for ratio of inner and outer radii $R_1/R_2 = 0.95$ with ratio of inner and outer angular speeds of the cylinder $\Omega_1/\Omega_2 = 0$ was obtained. Takhar, et al (1992) carried out an investigation on the effects of radial temperature gradient and axial magnetic field on the stability of Couette flow and found that temperature gradient affects the stability of the flow and simple axisymmetric instability occurs if the magnetic field is purely axial. It was also reported that in an ideal magnetohydrodynamics (MHD) fluid, the maximum growth rate is independent of the field strength. Recently an excellent investigation on the effect of axisymmetric stability on magnetorotational instability (MRI) of dissipative Couette flow was re-examined by Goodman and JI (2002) with emphasis on flows that would be hydrodynamically stable according to Rayleigh's criterion. This in comparison to the findings of Velikhov (1959) and Chandrasekhar (1960) that turbulence and orbital decay are driven by magnetorotational instabilities. The former also observed that exchange of stability appears to occur through marginal nodes but magnetic eigen functions are smooth and obey a fourth order differential equation in the inviscid limit. In all these works, studies of the attendant fluid flow characteristics in a porous media have been scanty and the existing investigations have often taken the assumption of a fully developed flow so that only a single space co-ordinate is involved in the analysis, Raptis et al (1981a).

This poses a fundamental linkage problem from the standpoint of basic research in stable fluid flow hence the study of the fluid flow in free space and in Cartesian coordinate system. Though radiation is been considered as reported by many research papers, permeability has often been neglected and little attention was given to the combined effect of the

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presence of permeability and radiation that are of interest in Structural engineering, Petroleum engineering, Geophysics and Geology just to mention but a few. It is reported that both permeability and radiation has independently destabilizing effect on the stability of Couette flow. 0ur goal is to examine the stability or otherwise 0f the presence of radiation and permeability on couette flow phenomena.

MATHEMATICAL FORMULATION

We consider the two dimensional steady heat flow of fluid in a horizontal porous medium in the Cartesian coordinate system such that t is the time. If u is the velocity component, then the equation of continuity and Navier-Stokes as well as energy is given by;

$$\frac{\partial \rho}{\partial t} = -\nabla.\rho u \quad (1)$$

$$\rho \frac{\partial \rho}{\partial t} = -\nabla P + \mu \nabla^2 u + \rho g \quad (2)$$

$$\frac{dT}{dt} = a^2 \nabla^2 T \quad (3)$$

Where P is pressure, ρ is fluid density, g is acceleration due to gravity, μ is absolute viscosity, T is temperature, $a^2 = \frac{K}{\rho c_v}$ is thermal diffusivity and ∇ is a del operator.

Considering the problem of a horizontal layer of fluid being heated from below, for us to investigate the effect of permeability and radiation on the stability of the system, we insert $\frac{\vartheta}{k} u$ and $\delta(T - T_0)$ where ϑ is kinematics viscosity, k is permeability of the medium under consideration, $\delta = 4\int_0^\infty \left(\alpha_{k^*} \frac{\partial B}{\partial T} \right) dk^*$. B is Plank's function, α_{k*} is absorption coefficient,

K* is frequency of radiation in equation (2) and (3) respectively, we get

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \rho g - \frac{\vartheta}{k} u \quad (4)$$

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + \delta (T - T_0) \quad (5)$$

Perturbation: Denoting the disturbance in the velocity, temperature and pressure field by

$$U = U_0 + U^1,\ T = T_e + T^1, \text{ and } T = P_e + P^1, \quad (6)$$

Where subscript e denotes equilibrium position. Substituting equation (6) into equations (1), (4) and (5) and neglecting all items that may involve products and squares of perturbation quantities, we obtain the following linearized equations.

$$\frac{\partial u^1}{\partial x} = 0 \quad (7)$$

for fluid at constant density (ρ)

$$\rho \frac{\partial u^1}{\partial t} = \frac{\partial P^1}{\partial x^1} + \mu \frac{\partial^2 u^1}{dx^{12}} + \rho g - \frac{\vartheta}{k} u^1 \quad (8)$$

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T^1}{\partial x^{12}} + \delta \left(T^1 - T^1{}_0 \right) \quad (9)$$

In steady state, equation (8) and (9) can be written as;

$$-\frac{\partial P^1}{\partial x} + \mu \frac{\partial^2 u^1}{dx^{12}} + \rho g - \frac{\vartheta}{k} u^1 = 0 \quad (10)$$

$$a^2 \frac{\partial^2 T^1}{\partial x^{12}} + \delta \left(T^1 - T^1{}_0 \right) = 0 \quad (11)$$

Non- Dimensional Analysis: Using the following dimensional quantities

$$x = \frac{x^1}{d},\ P = \frac{P^1}{\rho u^2},\ \alpha = \frac{\delta d^2}{\vartheta}$$

$$k_0 = \frac{\vartheta \mu d^2}{kP},\ u = \frac{u^1}{V},\ \beta = \frac{a^2 \rho t}{T\infty},$$

$$g = \frac{gd}{u^2},\ \theta = \frac{T - T_0}{T - T_x},\ \mathrm{Re}^{-1} = \frac{\mu}{Vd},$$

equations (7), (10) and (11) can be re-written as

$$\frac{\partial u}{\partial x} = 0$$

$$-\frac{\partial P}{\partial x} + \mathrm{Re}^{-1}\frac{\partial^2 u}{dx^2} + \rho g - K_0 u = 0 \quad (12)$$

$$\beta \frac{\partial^2 \theta}{\partial x^2} + \alpha\theta = 0 \quad (13)$$

Analytical Solution: Following equation (13), we assume a solution of the form

$$\theta = Ae^{\lambda x} \quad (14)$$

and imposing the boundary condition, $\theta(0) = 0$ we arrive at the feasible solution as

$$\theta(x) = \sin\sqrt{\frac{\alpha}{\beta}}x \quad (15)$$

where B is constant of integration.

The solution of equation (12) can be determined if changes in fluid density depends only on temperature as illustrated by the Boussinesq approximation $\Delta\rho = -\rho_0 E(T - T_0)$ where; $\Delta\rho$ is little change in fluid density, ρ_0 is fluid density at some properly chosen T_0, T_0 is temperature at which $\rho = \rho_0$ and E is coefficient of volume expansion.

The Boussinesq approximation can take the form

$$\Delta\rho = -\rho_0 E\theta \quad (15a)$$

Substituting equations (15) and (15a) in (12) where ρ is the perturbed fluid density,

$$-\frac{\partial P}{\partial x} + \mathrm{Re}^{-1}\frac{\partial^2 u}{dx^2} + g\rho_0 EB\sin\sqrt{\frac{\alpha}{\beta}} - k_0 u = 0 \quad (16)$$

If we assume that $\frac{\partial P}{\partial x}$ is $-k_p$ (constant) a pressure gradient, also for

simplicity $\rho_0 EgB = Z$ and rearrangement, equation (16) can be written as

$$\frac{\partial^2 u}{\partial x^2} - \mathrm{Re}^{-1} k_0 u = \mathrm{Re}\, Z \sin\sqrt{\frac{\alpha}{\beta}}x - \mathrm{Re}\, k_p \quad (17)$$

The complete solution of equation (17) is

$$U(x) = C_1 Cosh\sqrt{\mathrm{Re}\,k_0}\,x + C_2 \sinh\sqrt{\mathrm{Re}\,k_0}\,x + \left(\frac{\mathrm{Re}Z}{-\alpha/\beta + \mathrm{Re}\,k_0}\right)\sin\sqrt{\frac{\alpha}{\beta}}x + \frac{k_p}{k_0} \quad (18)$$

The boundary condition is satisfied by the assumption that the fluid velocity at the well of the plates must be equal to the wall velocity. The boundary condition is therefore satisfied by U(0) = 0, U(d) = U. Imposing these boundary conditions in (18) results in

$$U(x)=-\frac{K_p}{K_0}Cosh\sqrt{\mathrm{Re}K_0}x+\left(u-\frac{\mathrm{Re}Z}{-\alpha/\beta+\mathrm{Re}K_0}-\frac{K_p}{K_0}\right)\sinh\sqrt{\mathrm{Re}K_0}x+\left(\frac{\mathrm{Re}Z}{-\alpha/\beta+\mathrm{Re}K_0}\right)\sin\sqrt{\frac{\alpha}{\beta}}x+\frac{K_P}{K_0} \quad (19)$$

For $K_p = 0$

$$U(x)=\left(U-\frac{\mathrm{Re}Z}{-\alpha/\beta+\mathrm{Re}K_0}\right)\sinh\sqrt{\mathrm{Re}K_0}x+\left(\frac{\mathrm{Re}Z}{-\alpha/\beta+\mathrm{Re}K_0}\right)\sin\sqrt{\frac{\alpha}{\beta}}x \quad (20)$$

Analysis into Normal Modes: We examine the stability of each of these modes individually following the method of Chandrasekhar (1963), Bestman (1988) and Opara (1989). For the problem at hand, the analysis can be made in terms of two dimensional periodic wave numbers. Thus we assign to all quantities describing the perturbation a dependence on z, y and t in the form

$$Exp\left[i\left(K_yY+K_zZ\right)+nt\right] \quad (21)$$

Where n is a time constant and $(K^2_y + K^2_z)^{½}$ is given as K^1, the resultant wave number of the disturbance.

Also, introducing the following non- dimensional variables $a = kd$, σ = n ^{d2} $_k$

We write;

$$(U,\theta,P)=(U(x),\theta(x),P(x))\exp[i(K_yY+K_zZ)+nt]$$

$$\nabla^2=\frac{\partial}{\partial x^2}-K^2,-K^2=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},D=\frac{d}{dx} \quad (22)$$

Substituting equation (22) into equations (1), (4) and (5), following Hocking (1958), Bestman and Opara (1990) and eliminating pressure, we obtain;

$$\left[\sigma\frac{k}{\vartheta}-\left(D^2-a^2-K_0\right)\right]\left(D^2-a^2\right)U=-Ra^2\theta \quad (23)$$

and

$$\left[\sigma-\left(D^2-a^2-\frac{\alpha}{\beta}\right)\right]\theta=U \quad (24)$$

Coupling equation (23) and 24 following Opara (1994)

$$\left[\sigma\frac{k}{\vartheta}-\left(D^2-a^2-K_0\right)\right]\left(D^2-a^2\right)\left[\sigma-\left(D^2-a^2-\frac{\alpha}{\beta}\right)\right]\theta=-Ra^2\theta \quad (25)$$

a is the resultant dimensional wave number and R is $\frac{g\alpha Ed^4}{K\vartheta}$, the

Rayleigh number. To find the critical value of R as a function of a, we set σ = 0 equation (25) becomes;

$$\left[-\left(D^2-a^2-K_0\right)\left(D^2-a^2\right)\left(D^2-a^2-\frac{\alpha}{\beta}\right)\right]\theta=-Ra^2\theta \quad (26)$$

Following Opara (1994), by differentiating equations (23) and (24) even number of times, we successively conclude that all the even derivatives of U, θ must vanish for x = 0 or 1. Therefore, the proper solution for U, θ appropriate for the lowest mode is;

$$U = A\sin \pi x, \theta = A\sin \pi x \tag{27}$$

Substituting equation (27) in equations (23) and (24) we obtain after simplification.

$$\left[-\left(\pi^2 + a^2 + K_0\right)\left(D^2 + a^2\right)\left(\pi^2 + a^2 + \frac{\alpha}{\beta}\right)\right]\theta = -Ra^2 \tag{28}$$

RESULTS

K_p=2, K_o=1, x= 1-5, u=1, z=1.

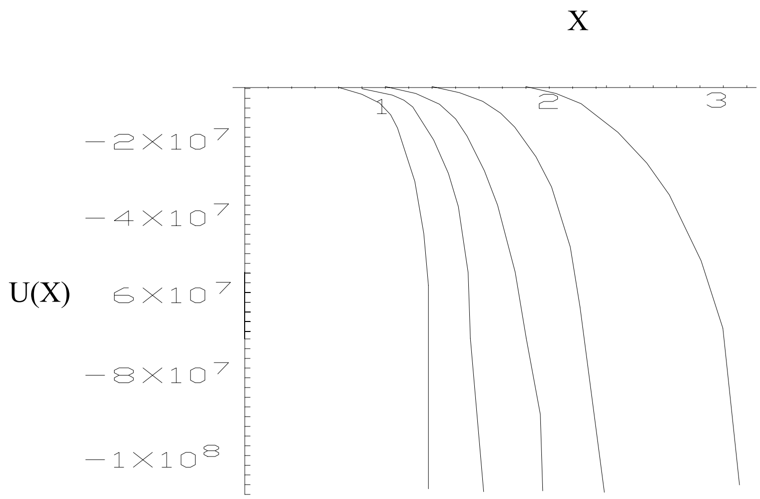

Fig. 1. The dependence of the velocity on both the direction x and the Reynolds' number Re

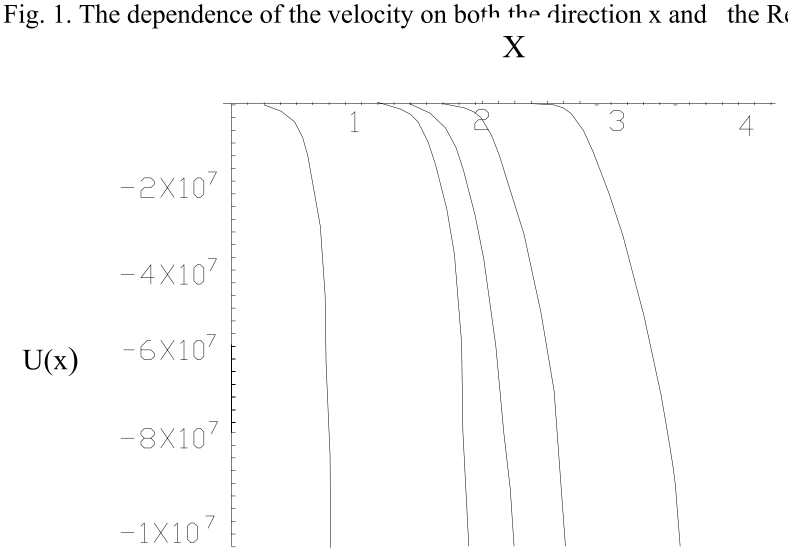

Fig. 2. Differential pressure kept at zero in figure 1.

Fig. 1 shows that at any fixed Reynolds' number, discrepancy exist in the mean curve of the theoretical description of Couette flow caused by the differential temperature and permeability .At Reynolds' number zero, there appears no curve on fig.2 which is in accord with those of Walowit et al (1964) for the stability of Couette flow at zero Reynolds' Number over the range of radius 0.9>N>0.1.

The solution of (13) is also in line with the findings of Takhar, Ali and Soundalgekar (1992) that temperature gradients affects the stability of Couette flow. Stability is maintained at small wave numbers regime ($0.1\leq a \geq 5$) with Ko = 0.2 and α = 0.4, however the situation is reversed at high wave number ($a \geq 10$) with Ko = 0.2 and radiation increased to 20.5. Also at high wave number with α = 0.4 and permeability increased up to 10 shows a considerable agreement with the work of NG and Turner (1981) and Takeuchi and Jankowski (1981).

Comparison of fig. 2 with the work of Hanson and Martin (1975) shows similarity in the shapes of the curves but complete reversal at very high Reynolds' number and low Reynold's number regime. However, we observed that at high Reynold's number (Re-2500) and wave number ($a > 10$), instability sets in as the fluid progresses in the presence of permeability and radiative heat.

Conclusion: In view of the geometry in which the fluid flows, porosity and differential temperature must be overcome to demonstrate stable flow of fluid. Equation (19) as a general case is a superposition of Couette flow and Poiseuile flow. The Couette flow was realized from equation (19) by setting $K_p = 0$. The fluid flow considered using the Cartesian coordinate system from our presentation, extends to infinity in the x direction and are two dimensional hence difficult to realize in application but being fundamental, it forms the basis and are often used as good approximations. If the geometry is cylindrical, then the cylindrical coordinate system is most suitable owing to the boundaries of the flow field because the axial direction of flow extends to infinity but changes in flow quantities in axial direction must be periodic so that these quantities do not take on infinite values at infinity.

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