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# Seasonal Variation of Power Distribution in Niger State of Nigeria using Markov Model with Non-Stationary Transition Probabilities

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**ABSTRACT**: This paper presents the application of Markov chain model with non-stationary transition probabilities to study the monthly data of the power distribution in Niger state in the wet, Dry-Hot and Hamatten/Dry-Hot seasons. The result indicates an optimal power distribution of over 150,000MWwith probability 0.49 during the wet season, 0.25 during the hot-dry season and 0.19 in the hot-cold season respectively. The variation of power distribution directly affects the electricity consumers. Markov chain model could be used as a predictive tool for determining the power distribution pattern at different seasons in the Study area. These predictions might be used for the management of (NCC) for effective distribution of megawatts.

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Electric power distribution is the final stage in the delivery of electric power, it carries electricity from the transmission system to individual consumers. Onochie, *et al* (2015) said that adequate power supply is the hallmark of a developed economy. Any nation whose energy need is epileptic in supply, prolongs her development and risks losing potential investors. Again, it is an unavoidable prerequisite to any nation's development. Electricity generation, transmission and distribution are the three stages of delivering electricity to consumers. The delivery of electricity to consumers in Nigeria has multidimensional problems (Sule, 2010) focused on capacity of electricity generation in Nigeria and the major factors affecting electricity generation, transmission and distribution in the country. The factors are none diversification of sources of energy used in electricity generation, poor maintenance culture, electrical power transmission line losses due to long distance between generating stations and load centers. Studies have been carried out on electricity consumption around the globe. Ubani, (2009) determined the electricity consumption pattern in south-south geopolitical region of Nigeria. The results showed that there were significant differences in electricity consumption pattern amongst the five states that constitute the south-south geopolitical region. River state had the highest mean of consumption rate, followed in descending order by Delta, Edo, Akwa Ibon and Bayelsa states. He recommended for strategic and systematic distribution of electricity to ensure adequate supply in south-south geopolitical region. Abubakar, (2005) studied Markov chain model with stationary and non-stationary transition probabilities for asthmas disease. The effect of treatment on the disease is also considered. The model could be used to study the condition of asthma patient in the Nigeria environment when the relevant data could be obtained. Mohammed, (2013) modeled the three state Markov model considered for both discrete and continuous time for the reservoir elevation. The effect of the varying season condition was also studied in the model. The model was used to analyse the data obtained from the dam. The result confirms that the reservoir elevation changes with the dry and wet seasons in Nigeria. It was observed also that the reservoir will on the long run, be in high elevation by 49% and 56% in the discrete time and continuous time respectively. Akyuz *et al.,* (2012) embarked on a study that use first and second order Markov chain models to dry and wet periods of annual stream flow series to reproduce the stochastic structure of hydrological droughts. In their study, they found that the second – order Markov (MC2) model in general gives results that are in better agreement with simulation results as compared with the first order (MC1) model. In view of these uncertainties, it is an

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attempt in this paper to formulate a Markov chain model that could be used to make prediction of the power distribution quantitatively.

## MATERIALS AND METHODS

*Study Area and Data Source:* The data used in this research work, were collected from the National Control Centre (NCC). In Nigeria, the electricity generating stations are interconnected to form the national grid at 330/132KV with a single National Control Centre (NCC) in Oshogbo and sub control centre at Shiroro (Minna) where power generated is being shared to eleven (11) distribution companies.

*Markov Chain:* Consider a random variable $X$ that is indexed by time parameter $n$, such a process is called stochastic process ( $X_n$ $n=0,1,2,\ldots$). Suppose that the stochastic process $X_n$ $n=0, 1,2,\ldots$ takes on a finite or countable number of possible values such that:

$$P\{X_{n+1}=j/X_n=i, X_{n-1}=i_{n-1}\ldots X_1=i_{11}\, X_0=i_0\}=P_{ij} \quad (1)$$

for all states $i_0, i_1,\ldots i_{n-1}, i, j$ and all $n \geq 0$. Such a stochastic process is known as a Markov chain (Ross 1996). Since probabilities are nonnegative and since the process must make a transition into some states, we have that

$$P_{ij} \geq 0, \quad i, j \geq 0; \quad \sum_{j=0}^{\infty} P_{ij} = 1, \quad i = 0,1,\ldots\ldots.$$

*Formulation of the Model:* It is a fact that the amount of power distributed and consumed on monthly basis is not a constant but changes every month unpredictably. It is therefore necessary in this paper to provide a fair prediction of power distribution with a Markov principle. The model also takes into consideration the changes in the power distribution during the raining period and dry period respectively. Let the amount of power distributed (MW) in a month be considered a random variable. The values of this collection of random variable are described by the following states:

STATE 1 Fairly Adequate Power Distribution (Greater or equal to 150,000MW): STATE 2 Moderate Power Distribution (Between130, 000MW and 150,000MW): STATE 3 Low Power Distribution (Between 110,000MW and130, 000MW): STATE 4 Scanty Power Distribution (Less than 110,000MW)
A possible transition between the states is given in figure 1

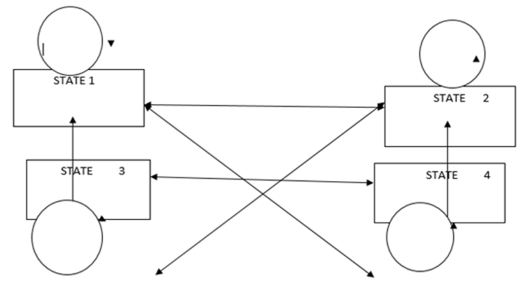

**Fig 1** State Transition Diagram for the Power Distribution

Let $P_{ij}$ be the probability that the power distribution presently in state $i$ will be in state $j$ in the next transition. Then

$$p = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \qquad 2$$

Equation (2) is the transition probability matrix for the power distribution. The above model shall be used to study the Monthly data recorded from distribution station.
Let
$P_n = \left(P_1^{\ n}, P_2^{\ n}, P_3^{\ n}, p_4^{\ n}\right)$
Denote the probabilities of finding power distribution in any of the state's 1, 2, 3,4 respectively, on $n$ period. Then, we have

$$P^n = P^{n-1}P \qquad (3)$$

On iteration we have

$$P^n = P^0 P^n \qquad n=1,2,3,4 \qquad (4)$$

Where $P^0$ is any starting vector of probabilities

*The Steady State Probabilities*: The process reaches a steady state after a sufficiently large period of time. This is the equilibrium probability distribution $\pi = (\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)$ and it is obtained by letting $n \to \infty$ in equation (3) Thus, we have

$$\pi = \pi P \qquad (5)$$

and the sum of the component of $\pi$ must be unity

$$i.e \qquad \sum_{i=1}^{4} \pi_i = 1$$

We use these last two equations to find the limiting state probabilities for the process.

*Modeling the Seasonal Effect:* Suppose that the probable causes and outcome of the power consumption change with the seasons. Let the following three seasons be considered as the transition time: (i) Dry-Hot season, (ii) Wet season (iii) Hamattan/Dry-Hot season

The period of Dry-hot season is from March to June, while the Wet season is from July to October and Hamatten/Dry-hot season is from November to February. Each of the seasons has its own transition count and transition probability matrices ( Jain 1988). We denote the transition count matrices and probabilities as: $M_1$ Transition count matrix for Dry-Hot season; $M_2$ Transition count matrix for Wet season; $M_3$ Transition count matrix for Hamatten/Dry-Hot season; $P_1$ Transition probability matrix for Dry-Hot season ; $P_2$ Transition probability matrix for Wet season ; $P_3$ Transition probability matrix for Hamatten/Dry-Hot season

Let $M_k = F_{ij}(k)$, $i, j$ =1,2,3,4and $k$ =*1,2,3* (6)

And $P_k = P_{ij}(k), i, j = 1,2,3,4 \; k = 1, 2,3$ (7)

$F_{ij}(k)$ Denotes the transition count from state $i$ to state $j$ for the season $k$ . $P_{ij}(k)$ is the transition probability from state $i$ to state $j$ for the season $k$

$$Accordingly, \quad P_{ij}(k) = \frac{F_{ij}(k)}{F_i(k)} \quad k = 1, 2,3 \; and \quad ij = 1, 2, 3,4 \tag{8}$$

Where $F_i = \sum_{j=1}^{4} F_{ij}(k)$

**Test for stationary of the probability Matrices $P_k$:** To test for independence of $P_k$ on *k*, the null hypothesis is stated thus:

$H_0 : P_{ij}(k) = P_{ij}$ for all $i, j = 1,2,3,4$ and for all $k$

$H_1 : P_{ij}(k)$ depends on $k$

The transition count matrix *M* is given by

$$M = \sum_{k=1}^{3} M_k = (F_{ij}) \tag{9}$$

Where $F_{ij} = \sum_{k=1}^{3} F_{ij}(k)$

The maximum likelihood estimate of the stationary transition probability matrix is

$$P_{ij} = \frac{F_{ij}}{F_i} \tag{10}$$

Where $F_i = \sum_{j=1}^{4} F_{ij}(k)$

The λ, the likelihood ratio criterion is given by

$$\lambda = \underset{ij=1}{\overset{4}{\pi}} \underset{k=1}{\overset{3}{\pi}} \left( \frac{p_{ij}}{p_{ij}(k)} \right)^{F_{ij}(k)} \quad 11)$$

$-2\ln\lambda = \chi^2_{m(m-1)(t-1)}$ (Bhat 1984) (12)

Where $m$ is the number of states and $t$ is the time parameter, we evaluate λ and calculate $-2In\lambda$ . We then get the critical value of $\chi^2$ at α significance level and compare it with $-2In\lambda$ it is then decided whether to accept or reject the null hypothesis. With the acceptance of $H_0$, we have a homogenous Markov chain model. The model is represented by a single transition count matrix (9) and $P_{ij}^{,s}$ are estimated from (10). Otherwise, we have non-homogeneous Markov chain model. The stochastic matrix $P$ can be written as

$$P = P_1 P_2 \tag{13}$$

And $P_{ij}^{,s}$ are estimated from (10)

The limiting state probability vectors $\pi_1$ $,\pi_2$ $and$ $\pi_3$ for the three seasons are then obtained from the following.

$$\pi_1 = \pi_0 p_1 \tag{14}$$

$$\pi_2 = \pi_1 P_2$$

$$\pi_3 = \pi_2 P_3$$

Where $\pi_0$ is the equilibrium transition probability matrix $P$

## RESULTS AND DISCUSSION

A summary statistics for 10 years for the monthly power distribution of Niger state is contained in table 1.

**Table 1: A Monthly summary of Megawatts Distribution in Niger State**

| CLASS INTERVAL (MW) | STATE | FREQUENCY |
|---|---|---|
| Greater or equal to150,000 | 1 | 36 |
| Between 130,000 and below 150,000 | 2 | 21 |
| 110,000 to 130,000 | 3 | 25 |
| Less than110,000 | 4 | 36 |
| Total | | 20 |

We provide the following transition count matrices and transition probability matrices using equations (1) and (2) respectively for the three (3) seasons.

$$M_1=\begin{bmatrix}4&5&3&0\\0&8&3&4\\3&0&0&3\\3&0&0&4\end{bmatrix} M_2=\begin{bmatrix}13&7&0&4\\3&0&3&0\\4&0&2&0\\2&2&1&1\end{bmatrix} M_3=\begin{bmatrix}0&0&0&1\\0&3&3&0\\0&3&9&0\\4&2&4&10\end{bmatrix} \quad (15)$$

$$P_1=\begin{bmatrix}0.3333&0.4167&0.2500&0\\0&0.5333&0.2000&0.2667\\0.5000&0&0&0.5000\\0.4286&0&0&0.5714\end{bmatrix}$$

$$P_2=\begin{bmatrix}0.5417&0.2914&0&0.1667\\0.5000&0&0.5000&0.\\0.8000&0&0.2000&0\\0.3333&0.3333&0.1667&0.1667\end{bmatrix}$$

$$P_3=\begin{bmatrix}03500&0.2500&0.2500&0.1500\\0&0.5000&0&0.5000\\0&0.7500&0.2500&0\\0.2000&0.1000&0.2000&0.5000\end{bmatrix}$$

Using equation (9) and (10), we have

$$M=\begin{bmatrix}24&17&8&8\\3&8&12&4\\7&3&11&3\\12&4&3&15\end{bmatrix} P=\begin{bmatrix}0.4210&0.2982&0.1404&0.1404\\0.1111&0.2963&0.4444&0.1481\\0.2917&0.1250&0.4583&0.1250\\0.3529&0.1176&0.0882&0.4412\end{bmatrix}$$
(16)

From equation (11), we have $\lambda = 0.0000000000 \quad 2314635$ ; for m=4, t= 3 -2In$\lambda$ = 42.6951827318

The critical value of $\chi^2_{24}$ at $\lambda$= 0.05 is 31.41 Since the calculated value of -2In$\lambda$ is greater than the critical value of $\chi^2_{24}$ at $\alpha$ significance level the null hypothesis of constant transition probability matrix is rejected. Then we have non- homogenous Markov chain model. Thus the maximum likelihood estimates of the transition probability matrix using equation (13) give us

$$P=\begin{bmatrix}0.2172&0.3951&0.2229&0.1647\\0.1893&0.4411&0.2256&0.1440\\0.1865&0.3447&0.1636&0.3051\\0.1813&0.3514&0.1628&0.3044\end{bmatrix} \quad (17)$$

$$P^2=\begin{bmatrix}0.1934&0.3948&0.2008&0.2108\\0.1928&0.3977&0.2020&0.2074\\0.1916&0.3894&0.1958&0.2232\\0.1914&0.3897&0.1959&0.2228\end{bmatrix}$$

$$P^8=\begin{bmatrix}0.1923&0.3935&0.1991&0.3214\\0.1923&0.3935&0.1991&0.2144\\0.1923&0.3934&0.1991&0.2143\\0.1923&0.3935&0.1991&0.2143\end{bmatrix}$$

$$P^{32}=\begin{bmatrix}0.1921&0.3951&0.1989&0.2142\\0.1921&0.3951&0.1989&0.2142\\0.1921&0.3951&0.1989&0.2142\\0.1921&0.3951&0.1989&0.2142\end{bmatrix} \quad (18)$$

$$P^{32}=\begin{bmatrix}0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\end{bmatrix}$$

Corrected to 2 decimal places

This is the probability of finding the power distribution in any of the four states for $n \geq 32$
From equation (5), the limiting state probability vector is given by

$$\pi = \pi P = [0.19 \quad 0.40 \quad 0.20 \quad 0.21]$$
$$\pi_0 = [0.19 \quad 0.40 \quad 0.20 \quad 0.21]$$

After 32 iterations using Maple software, equation (17) converged to equation (18). As $n \to \infty$ and using equation (4), we have

$$P^n = P^0 P^n = [1 \quad 0 \quad 0 \quad 0]\begin{bmatrix}0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\\0.19&0.40&0.20&0.21\end{bmatrix} = [0.19 \quad 0.40 \quad 0.20 \quad 0.21] \quad (19)$$

$$\pi_1 = \pi_0 P_1 = \begin{bmatrix}0.19 & 0.40 & 0.20 & 0.21\end{bmatrix}\begin{bmatrix}0.3333 & 0.4167 & 0.2500 & 0\\ 0 & 0.5333 & 0.2000 & 0.2667\\ 0.5000 & 0 & 0 & 0.5000\\ 0.4286 & 0 & 0 & 0.5714\end{bmatrix}$$

$$\pi_1 = \begin{bmatrix}0.25 & 0.29 & 0.13 & 0.33\end{bmatrix}$$

$$\pi_2 = \pi_1 P_2 = \begin{bmatrix}0.25 & 0.29 & 0.13 & 0.33\end{bmatrix}\begin{bmatrix}0.5417 & 0.2914 & 0 & 0.1667\\ 0.5000 & 0 & 0 & 0.5000\\ 0.8000 & 0 & 0.2000 & 0\\ 0.3333 & 0.3333 & 0.1667 & 0.1667\end{bmatrix}$$

$$\pi_2 = \begin{bmatrix}0.49 & 0.18 & 0.23 & 0.10\end{bmatrix}$$

$$\pi_3 = \begin{bmatrix}0.49 & 0.18 & 0.23 & 0.10\end{bmatrix}\begin{bmatrix}0.3500 & 0.2500 & 0.2500 & 0.1500\\ 0 & 05000 & 0 & 0.5000\\ 0 & 0.7500 & .25000 & 0\\ 0.2000 & 0.1 & 0.2000 & 0.5000\end{bmatrix}$$

$$\left.\begin{aligned}\pi_3 &= \begin{bmatrix}0.19 & 0.40 & 0.20 & 0.21\end{bmatrix}\\ \pi_1 &= \begin{bmatrix}0.25 & 0.29 & 0.13 & 0.33\end{bmatrix}\\ \pi_2 &= \begin{bmatrix}0.49 & 0.18 & 0.23 & 0.10\end{bmatrix}\\ \pi_3 &= \begin{bmatrix}0.19 & 0.40 & 0.20 & 0.21\end{bmatrix}\end{aligned}\right\} \quad (20)$$

This paper considered Markov model to study the power distribution in Niger state in discrete time. For the discrete time case, attempt was made to consider the seasonal variation in Nigeria. The result indicates that in the dry-hot season the power distribution shall be 0.25, 0. 29, 0.13 and 0.33 for fairly adequate power distribution, moderate power distribution, lower power distribution and scanty power distribution respectively in the long run. While, in the wet season with power distribution of about 0.49, 0.18, 0.23 and 0.10 for fairly adequate power distribution, moderate power distribution, lower power distribution and scanty power distribution respectively in the long run and also for the Hamatten/dry-hot season we have power distribution of about 0.19, 0.40, 0.20 and 0.21 for fairly adequate power distribution, moderate power distribution, lower power distribution and scanty power distribution respectively in the long run. The result indicates that the non-homogeneous Markov chain model has an optimal power distribution of over 150,000MW per month with optimal probability of 0.49 during the wet season. This is however followed by power distribution of 130,000MW to less than 150,000MW per month with an optimal probability of 0.40 during the hot-cold hamatten period. The least power (less than 110,000MW per month), distribution is expected during the Dry-hot season with maximum probability of 0.33. This is a very important result, confirming the experience of electrical consumers in Niger state. Relatively, there is a good power supply during the raining period when the Dam is filled up with water and the generation is at its peak. Nevertheless, during the hamatten cold-dry season, the supply of electricity will start to run down and get worse during the Hot-Dry period when the Dam almost get dried up and turbines are totally or partly shot down.

Since optimal power distribution of over 150,000MW is not obtainable in all the seasons, it is necessary to seek alternative power source such as solar and coal to make up for the short fall. The model could be used as a predictive tool for studying power distribution in Niger state. The prediction is information that could be useful for the (NCC)) and staff of distribution company for proper distribution of megawatts to states.

*Conclusion:* A stochastic model to analyze and predict monthly power distribution of Niger state has been presented. The stochastic model was formulated based on the principle of Markov chain. The result shows that power allocation of over 150,000MW with probability 0.49 is obtained during the wet season, 0.25 during the hot-dry season and 0.19 in the Hamatten/Dry-Hot season respectively in Niger state. The results from the model are an important information, that could assists the (NCC)) and staff of distribution company for proper distribution of megawatts to states.

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